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Unit – II

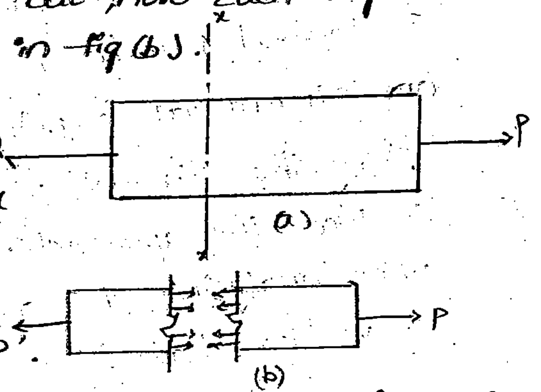
Principle stress & strains and theory of failures

Stress:

The applied external forces on a body are transmitted to supports through the material of the body. This phenomena tends to deform the body and causes it to develop equal and opposite internal forces. These internal forces by the verture of cohesion b/w the particles of materials which tends to resist the deformation. The magnitude of these internal forces are equal to the applied forces but in opposite direction.

let us consider a member which tends to pull at an intensity "P" as shown in fig(a), at the Sec x-x the member is cut, now each segment can be seen as shown in fig(b).

Each segment in fig(b) is in equilibrium under the action of internal forces resisting the external applied force 'P'.



This resisting force per unit area on the surface is turmed as intensity of stress. It is denoted by σ

$$\therefore \sigma = \frac{P}{A} \ N/mm^2$$

Strain:

The deformation of a body under load is propotional to it's length to study the behaviour of materials it is convenient to study the deformation of body per unit length, that it's total deformation. This length alonged per unit length of a body is called strain

$$\therefore \epsilon = \frac{\text{change in length}}{\text{original length}} = \frac{\delta l}{l}$$

principle plain and principle stresses

The plains which have no shear stresses are known as principle plains.

These plains carrying only normal stress, so the plains which carrying only normal stresses are known as principle stresses.

Stress analysis:

The stress varies from point to point on a loaded member, therefore the equilibrium of an eliment at a point is to be considered by taking the eliment of infinite decimal dimension, so that the approaching of the stress are easy while analysing the stress system the general conventions have been taken as follows.

1. Tensile stress is +ve, & compressive stress is -ve.

2. A pair of shear stress on parallel plains forming a clockwise couple is +ve and a pair of with counter clockwise is -ve.

3. Clockwise angle is +ve and anticlock wise angle is -ve.

→ For stress analysis the following cases are considered

1. Direct stress condition.
2. Biaxial stress condition
3. pure shear stress condition
4. Biaxial and shear stress condition.

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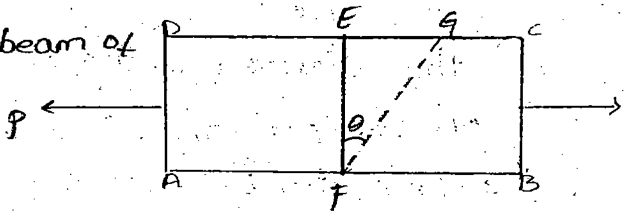
Methods of determining stress analysis

1. Analytical method
2. Graphical method.

Analytical method:

* member subjected to direct stresses in ⊥ plane

Consider a $\square^{le}$ beam of uniform c/s area 'A' with unit thickness.



Let 'p' be the axial force acting on the member.

Now stress on $AD = \frac{P}{A}$

$BC = \frac{P}{A}$

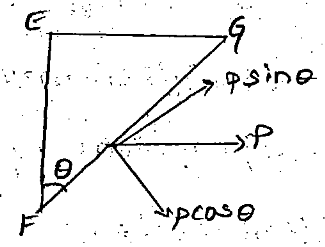
Consider a c/s EF ⊥r to line of action of force.

Now, the stress of EF can be determined by $= \dfrac{\text{Force}}{\text{Area of EF Section}}$

$$= \frac{P}{A \times t} = \frac{P}{A} \quad (\text{Say } t = 1)$$

We can clearly observe that the stresses acting on EF similar to stresses acting on AD & BC which are normal stresses and no shear stresses are acting on EF.

let us consider a point 'G' on the plane CD Connect F & G. The section FG making an angle 'θ' with section EF.



from $\triangle^{le}$ EFG.

$$\cos\theta = \frac{EF}{FG}$$

$$FG = \frac{EF}{\cos\theta}$$

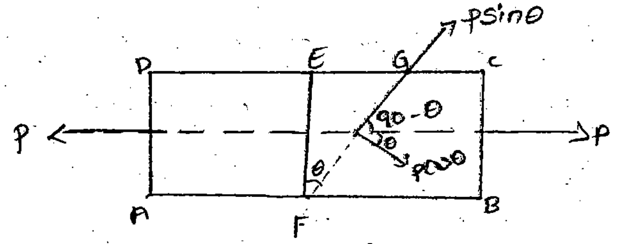
but $EF = A$

$$\therefore FG = \frac{A}{\cos\theta}$$

the stresses acting on section 'FG',

on Sec 'FG' the force making an angle θ, so the force can be resolved into 2 components.

1. Normal to Section i.e., $p\cos\theta$ or P_n
2. Tangential to section i.e., $p\sin\theta$ or P_t



σ_n = normal stress on section FG

$$\sigma_n = \frac{\text{Force normal to sec FG}}{\text{Area of sec FG}}$$

$$= \frac{P\cos\theta}{\frac{A}{\cos\theta}}$$

$$= \frac{P}{A}\cos^2\theta$$

$$\sigma_n = \sigma\cos^2\theta$$

σ_n is max when $\cos^2\theta$ is max

$\cos^2\theta$ is max when $\theta = 0$

$$\sigma_n = \frac{P}{A}, \text{ when } \theta = 0$$

which means that the plane normal to the axis of loading will clearly the max stress

σ_t = Tangential stresses on sec FG

$$\sigma_t = \frac{\text{Tangential force on sec FG}}{\text{Area of sec FG}}$$

$$= \frac{P\sin\theta}{\frac{A}{\cos\theta}}$$

$$= \sigma\cos\theta\sin\theta$$

$$= \frac{\sigma}{2}\, 2\sin\theta\cos\theta$$

$$\sigma_t = \frac{\sigma}{2}\sin 2\theta$$

σ_t is max when $\sin 2\theta$ is max

$\sin 2\theta$ is max when $\theta = 45^\circ$ or 135°

$\therefore \sigma_t = \frac{\sigma}{2}$

1. A rectangular bar of c/s area 10,000 mm² is subjected to an axial force of 20 kN. Determine the normal & shear stresses on the sec which is inclined at an angle 30° with normal c/s of bar?

Sol:

Given,

C/s area = 10,000 mm²

load $p = 20$ kN, $\theta = 30^\circ$

$$\sigma = \frac{P}{A} = \frac{20 \times 10^3}{10,000} = 2 \text{ N/mm}^2$$

$\sigma_n = \sigma \cos^2\theta$

$= 2 \times \cos^2 30$

$= 1.5$ N/mm²

$\sigma_t = \frac{\sigma}{2} \sin 2\theta$

$= \frac{2}{2} \sin 2(30)$

$\sigma_t = 0.867$ N/mm²

2. Find the dia of c/r bar which is subjected to an axial pull of 160 kN if the max allowable shear stress on any section is 65 N/mm²

Sol:

Given,

Axial load $p = 160$ kN

max shear stress $\sigma_t = 65$ N/mm²

w.k.t $\sigma_t = \frac{\sigma}{2}$

$65 = \frac{\sigma}{2}$

$\sigma = 130$ N/mm²

we know, $\sigma = \frac{P}{A}$

$130 = \frac{P}{A}$ ($\because P = 160$)

$A = 1230.76$ mm²

[$\because$ Area of c/r $= \frac{\pi d^2}{4}$]

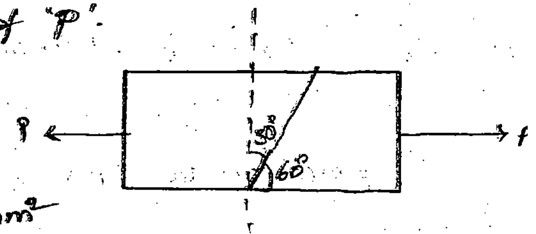
$\frac{\pi d^2}{4} = 1230.76$

$d = 39.58$ mm

$d \approx 40$ mm.

3. A rectangular bar of c/s area 11,000 mm² is subjected to a tensile load of 'p' as shown in fig. The permissible normal & shear stresses on the oblique section 'BC' are 7 N/mm², 3.5 N/mm². Determine the value of 'P'.

Sol:



Given, $A = 11,000$ mm²

$\sigma_n = \sigma \cos^2\theta = 7$ N/mm²

$\sigma_t = \frac{\sigma}{2} \sin 2\theta = 3.5$ N/mm²

$\theta = 30^\circ$

$\sigma_n = \sigma \cos^2 30 = 7$

$\sigma = 9.3333$

$\frac{P}{A} = \sigma$

$p = 102.6$ kN.

$\sigma_t = \frac{\sigma}{2} \sin 2\theta$

$3.5 = \frac{P}{22,000}$

$P = 88.9$ kN

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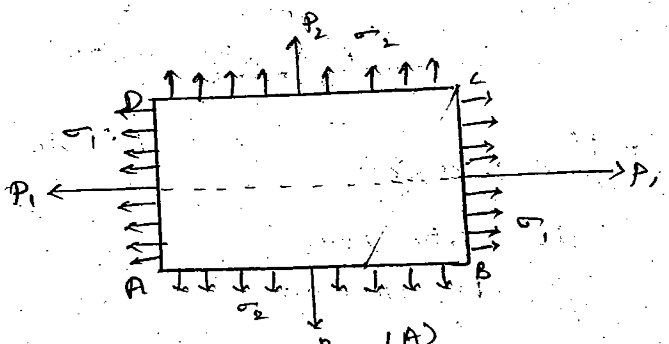
Member Subjected to Direct stress in 2 mutually perpendicular directions :-



(A)

let us consider a rectangle beam ABCD of uniform c/s area 'A' with unit thickness subjected to 2 direct tensile stresses as shown in fig (A).

let, σ_1 be the major stresses on AD & BC due to tensile force P_1

σ_2 be the Minor stresses on AB & CD due to tensile force P_2.

Force on BC (P_1) = stress * Area

$= \sigma_1 * BC * \text{thickness}$

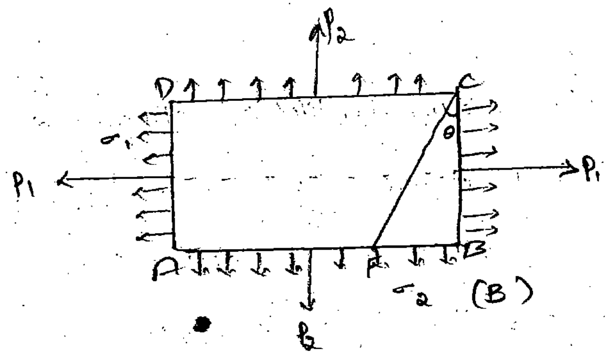
$= \sigma_1 * A$.

Force on AD $(P_2) = \sigma_2 * A$.

let, us consider a point F on plane AB and combined CF.

→ the section CF making an angle θ with the section BC.

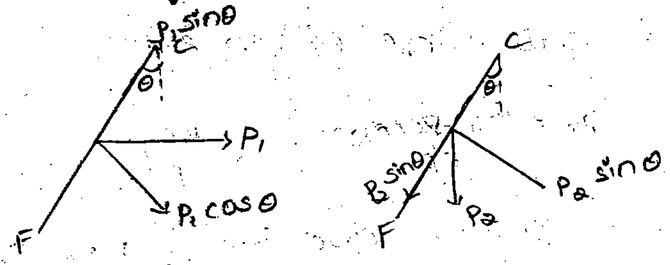
As shown in fig 'B'.



(B)

→ on section CF, the tensile forces are also acting where P_1 is acting is axial direction and P_2 is acting in downward direction.

These tensile forces are inclined to the section CF, so the P_1 and P_2 are resolved into components as shown in figs 'C' and 'D'



(C)

let, P_n = total normal forces on section CF, $= P_1 \cos\theta + P_2 \sin\theta$

But, $P_1 = \sigma_1 * BC$

$P_2 = \sigma_2 * BF$

$\therefore P_n = \sigma_1 BC \cos\theta + \sigma_2 * BF \sin\theta$

Let, P_t = total tangential forces along sec CF

$P_t = P_1 \sin\theta - P_2 \cos\theta$

$\therefore P_t = \sigma_1 BC \sin\theta - \sigma_2 BF \cos\theta$

Now,

σ_n = total normal ~~forces~~ stresses acting along section CF

$$\sigma_n = \frac{\text{total normal stresses along s/c CF}}{\text{Area of s/c CF}}$$

$$= \frac{\sigma_1 BC\cos\theta + \sigma_2 BF\sin\theta}{CF * 1}$$

$$\therefore \sigma_n = \sigma_1 \frac{BC}{CF}\cos\theta + \sigma_2 \frac{BF}{CF}\sin\theta \quad \text{—①}$$

But from Δ^{le} CFB,

$$\cos\theta = \frac{BC}{FC} \qquad \therefore \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin\theta = \frac{BF}{FC} \qquad \therefore \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

sub these eq^{ns} in eq^n ①

$$\sigma_n = \sigma_1 \frac{BC}{CF} + \sigma_2 \left(\frac{BF}{CF}\right)$$

$$\sigma_n = \sigma_1 \cos\theta(\cos\theta) + \sigma_2 \sin\theta(\sin\theta)$$

$$\therefore \sigma_n = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta$$

$$= \sigma_1\left(\frac{1+\cos 2\theta}{2}\right) + \sigma_2\left(\frac{1-\cos 2\theta}{2}\right)$$

$$= \frac{\sigma_1}{2} + \frac{\sigma_1 \cos 2\theta}{2} + \frac{\sigma_2}{2} - \frac{\sigma_2 \cos 2\theta}{2}$$

$$= \left(\frac{\sigma_1+\sigma_2}{2}\right) + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta \quad \text{—②}$$

σ_t = total tangential stresses acting on CF

$$\sigma_t = \frac{\text{total tangential forces along sec CF}}{\text{Area of CF}}$$

$$= \frac{\sigma_1 BC\sin\theta - \sigma_2 BF\cos\theta}{CF}$$

$$\therefore \sigma_t = \sigma_1 \frac{BC}{CF}\sin\theta - \sigma_2 \frac{BF}{CF}\cos\theta \quad \text{—③}$$

from Δ^{le} CFB

$$\cos\theta = \frac{BC}{FC}$$

$$\sin\theta = \frac{BF}{FC}$$

$$\sigma_t = \sigma_1 \cos\theta\sin\theta - \sigma_2 \sin\theta\cos\theta$$

$$= \frac{\sigma_1}{2} 2\sin\theta\cos\theta - \frac{\sigma_2}{2} 2\sin\theta\cos\theta$$

$$= \frac{\sigma_1}{2}\sin 2\theta - \frac{\sigma_2}{2}\sin 2\theta$$

$$\therefore \sigma_t = \left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta$$

Examples

* The resultant stresses on section CF are given by $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$

1. The tensile stresses at a point across 2 mutual ⊥r directions are 120 N/mm² and 60 N/mm². Determine the normal stresses, tangential stresses and resultant stresses on a plane inclined to minor axis at an angle of 30°.

Sol:

$\sigma_1 = 120\ N/mm^2$

$\sigma_2 = 60\ N/mm^2$

$\therefore\ \theta = 30°$

$$\sigma_n = \left(\frac{\sigma_1+\sigma_2}{2}\right) + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta$$

$$= \left(\frac{120+60}{2}\right) + \left(\frac{120-60}{2}\right)\cos 2(30°)$$

$\therefore\ \sigma_n = 105\ N/mm^2$

$$\sigma_t = \left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta$$

$$\sigma_t = \left(\frac{120-60}{2}\right)\sin 2\theta$$

$$= 25.98\ N/mm^2$$

$$\sigma_R = \sqrt{105^2 + 25.98^2}$$

$$= 108.16\ N/mm^2$$

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Obliquity

The angle that is made by the resultant stresses with the normal stresses is known as obliquity. It is denoted by 'ϕ'.

$\therefore \tan\phi$ can be written as $\frac{\sigma_t}{\sigma_n}$

Biaxial

Example:-

1. The stresses at a point in a bar are 200 N/mm^2 tensile and 100 N/mm^2 compressive. Determine the resultant stresses in magnitude & direction on a plane inclined at 60° to the axis of major stresses. Also determine the max. intensity of shear stress in a material on that plane.

analysis
tension is +ve
comp is −ve

Sol:

Given data.

$\sigma_1 = 200\ N/mm^2$ (tensile)

$\sigma_2 = -100\ N/mm^2$ (Compresion)

$\theta = 60°$ with major stress

$\sigma_R = ?$

$$\sigma_n = \left(\frac{\sigma_1+\sigma_2}{2}\right) + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta$$

$$= \left(\frac{200+100}{2}\right) + \left(\frac{200+100}{2}\right)\cos 2(60°)$$

$\sigma_n = \cancel{12}25\ N/mm^2$

$$\sigma_t = \left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta$$

$$= \left(\frac{200+100}{2}\right)\sin 2(60°) = 143.3012 \text{ (or)}$$

$$= 129.90\ N/mm^2$$

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

$\sigma_R = 132.2875\ N/mm^2$

max shear stress is obtained when

$\theta = 45°$ or $135°$

$\therefore \sigma_t = \frac{\sigma_1 - \sigma_2}{2}$

$= 150\ N/mm^2$

2. At a point in a strained material the principle tensile stresses across 2 mutual perpendicular directions are $80\ N/mm^2$ & $40\ N/mm^2$. Determine the normal stress, tangential stresses and resultant stress on a plane inclined at $20°$ with the major principle plane. Determine the obliquity also.

Sol:

Given data:

$\sigma_1 = 80\ N/mm^2$

$\sigma_2 = +40\ N/mm^2$

$\theta = 20°$ with major principle plane

$\sigma_R = ?$ $\quad\phi = ?$

$\sigma_n = ?$

$\sigma_t = ?$

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cos 2\theta$$

$$= \left(\frac{80 + 40}{2}\right) + \left(\frac{80 - 40}{2}\right)\cos 2(20°)$$

$= 75.96\ N/mm^2$

$$\sigma_t = \left(\frac{\sigma_1 - \sigma_2}{2}\right)\sin 2\theta$$

$$= \left(\frac{80 - 40}{2}\right)\sin 2(20)$$

$= 12.85$

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

$= 77.04\ N/mm^2$

Obliquity,

$$\tan\phi = \frac{\sigma_t}{\sigma_n}$$

$$\phi = \tan^{-1}\left(\frac{12.85}{75.96}\right)$$

$\phi = 9°40'$

* Member Subjected to pure Shear Stresses

Consider a rectangular bar ABCD of uniform cross sectional area 'A' with unit thickness.

→ let CF be the oblique plane making an angle 'θ' with the BC section

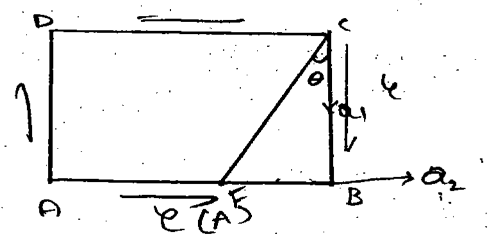
⇒ And τ be the shear stress acting on the plane BC & AD.

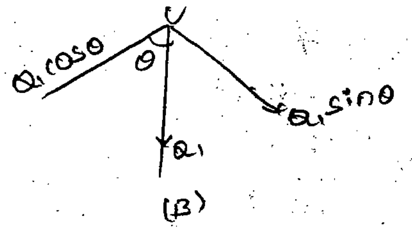
Q_1 = Shear force on BC

Q_2 = Shear force on BF

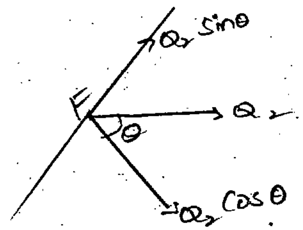
Q_1 = shear stress * A of BC, Q_2 = shear stress * A of FB
$= \tau\, BC * 1$ $\quad = \tau\, FB * 1$

the shear forces Q_1 and Q_2 are inclined with the oblique plane so, these are resolved into 2 components as shown in fig (B) and fig (C).





(B)



let P_n = total normal forces on s/c CF

P_t = total tangential forces on s/c CF

σ_n = total normal stresses on s/c CF

σ_t = total tangential stresses on s/c CF

$\therefore P_n = Q_1 \sin\theta + Q_2 \cos\theta$
$= \tau\, BC \sin\theta + \tau\, BF \cos\theta$

$\therefore P_t = Q_2 \sin\theta - Q_1 \cos\theta$
$= \tau\, FB \sin\theta - \tau\, BC \cos\theta$

$\therefore \sigma_n = \dfrac{\text{Total normal force s/c CF}}{\text{Area of s/c CF}}$

$= \dfrac{\tau BC \sin\theta + \tau BF \cos\theta}{CF}$

From △ABF
$\therefore \cos\theta = \dfrac{BC}{CF}$
$\therefore \sin\theta = \dfrac{BF}{CF}$

$\therefore \sigma_n = \tau \dfrac{BC}{CF} \sin\theta + \tau \dfrac{BF}{CF} \cos\theta$

$= \tau \cos\theta \sin\theta + \tau \sin\theta \cos\theta$

$= 2\tau \cos\theta \sin\theta$

$= \tau \sin 2\theta$

$\therefore \sigma_t = \dfrac{\text{Total tangential force}}{\text{Area of s/c CF}}$

$= \dfrac{\tau FB \sin\theta - \tau BC \cos\theta}{CF}$

$= \tau \dfrac{FB}{CF} \sin\theta - \tau \dfrac{BC}{CF} \cos\theta$

$= \tau \sin^2\theta - \tau \cos^2\theta \quad [\sin^2\theta - \cos^2\theta = -\cos 2\theta]$

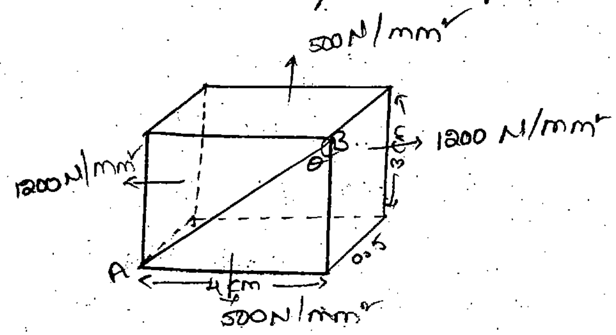
$= \tau (\sin^2\theta - \cos^2\theta)$

$\therefore \sigma_t = -\tau \cos 2\theta$

Here the -ve sign indicates downward direction

Example :-

A Small block 4cm long , 3cm height and 0.5 cm thickness . It is subjected to uniformally distributed tensite force 2 mutually ⊥r directions of 1200 N & 500 N as shown in fig . Compute the normal and shear stresses developed along the diagonal AB.



Sol:

Given data

Length (l) = 4cm

height (h) = 3cm

thickness (t) = 0.5 cm.

$\sigma_1 = 1200\ N/mm^2$

$\sigma_2 = 500\ N/mm^2$

From fig., $\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\theta = 53^\circ 7'$

$\sigma_n = \sin 2\theta$

$\sigma_n = \left(\frac{\sigma_1+\sigma_2}{2}\right) + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta$

$= \left(\frac{1200+500}{2}\right) + \left(\frac{1200-500}{2}\right)\cos 2(53^\circ 7')$

$= 752.16\ N/mm^2$

$\sigma_t = \left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta$

$= \left(\frac{1200-500}{2}\right)\sin 2(53^\circ 7')$

$= 336.04\ N/mm^2$

$\sigma_R = 823.815\ N/mm^2$

$Stress = \frac{Force}{Area}$

$= \frac{1200}{4 \times 3}$

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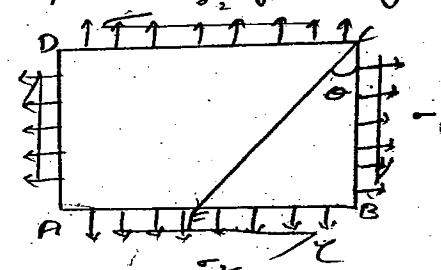
*Member subjected to Biaxial and shear stress on an oblic plane

Consider a rectangular bar with uniform c/s area A with unit thickness is subjected to a tensile stresses (σ_1, σ_2) and shear stress as show in fig

let, CF be an oblique plane making an angle 'θ' with the normal plane BC



let, P_1 = ~~tangential~~ tensile force due to ~~tangential~~ tensile stress σ_1 on BC

P_2 = ~~tangential~~ tensile force on BF due to tensile stress σ_2

Q_1 = Shear force on 'BC' due to shear stress (τ)

Q_2 = shear force on BF due to shear stress.

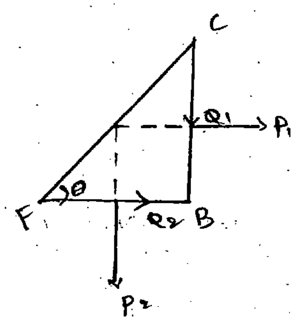
P_n = total normal forces on oblique section CF

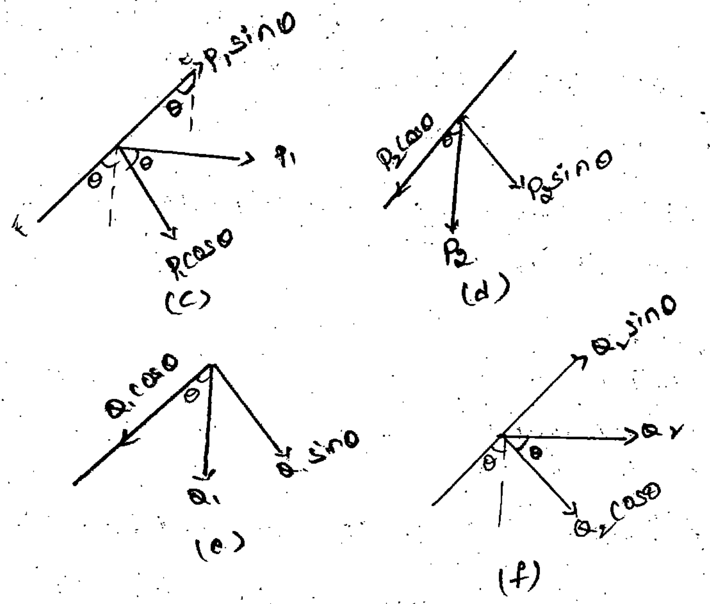
P_t = total tangential forces on oblique section CF

σ_n = normal stress on CF

σ_t = tangential stress on CF

30/11/18 The forces acting on s/c CF P_1, P_2, Q_1 and Q_2 are inclined with the oblique plane, so these forces are resolved into Components as shown in fig (c), (d), (e) & (f)





$$P_n = P_1\cos\theta + P_2\sin\theta + Q_1\sin\theta + Q_2\cos\theta$$

But, P_1 = stress on s/c BC * Area of s/c BC

$= \sigma_1 * BC * 1$

$\therefore P_1 = \sigma_1 BC$

P_2 = Stress on s/c BF * Area of s/c BF

$= \sigma_2 * BF * 1$

$\therefore P_2 = \sigma_2 BF$

Q_1 = shear stress on s/c BC * Area of s/c BC

$\therefore Q_1 = \tau BC * 1 = \tau BC$

Q_2 = shear stress on s/c BF * Area of s/c BF

$= \tau BF * 1$

$\therefore Q_2 = \tau BF$

Now,

$$P_n = \sigma_1 BC\cos\theta + \sigma_2 BF\sin\theta + \tau BC\sin\theta + \tau BF\cos\theta$$

$$\sigma_n = \frac{\text{Total normal force on s/c CF}}{\text{Area of s/c CF}}$$

$$= \frac{\sigma_1 BC\cos\theta + \sigma_2 BF\sin\theta + \tau BC\sin\theta + \tau BF\cos\theta}{CF}$$

$$\sigma_n = \sigma_1\frac{BC}{CF}\cos\theta + \sigma_2\frac{BF}{CF}\sin\theta + \tau\frac{BC}{CF}\sin\theta + \tau\frac{BF}{CF}\cos\theta$$

$\therefore$ from Δ^{le} CFB,

$\cos^2\theta = \frac{1+\cos2\theta}{2}$

$\sin^2\theta = \frac{1-\cos2\theta}{2}$

$\cos\theta = \frac{BC}{CF}$

$\sin\theta = \frac{BF}{CF}$

$$\therefore \sigma_n = \sigma_1\cos^2\theta + \sigma_2\sin^2\theta + \tau\cos\theta\sin\theta + \tau\sin\theta\cos\theta$$

$$= \sigma_1\cos^2\theta + \sigma_2\sin^2\theta + 2\tau\cos\theta\sin\theta$$

$$= \sigma_1\left(\frac{1+\cos2\theta}{2}\right) + \sigma_2\left(\frac{1-\cos2\theta}{2}\right) + \tau\sin2\theta$$

$$= \frac{\sigma_1}{2} + \sigma_1\frac{\cos2\theta}{2} + \frac{\sigma_2}{2} - \frac{\sigma_2\cos2\theta}{2} + \tau\sin2\theta$$

$$\sigma_n = \left(\frac{\sigma_1+\sigma_2}{2}\right) + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos2\theta + \tau\sin2\theta$$

$P_t = P_1 \sin\theta - P_2 \cos\theta - Q_1 \cos\theta + Q_2 \sin\theta$

$P_t = \sigma_1 BC \sin\theta - \sigma_2 BF \cos\theta - \tau BC \cos\theta + \tau BF \sin\theta$

$\therefore \sigma_t = \sigma_1 \frac{BC}{CF} \sin\theta - \sigma_2 \frac{BF}{CF} \cos\theta - \tau \frac{BC}{CF} \cos\theta + \tau \frac{BF}{CF} \sin\theta$

$\sigma_t = \sigma_1 \cos\theta \sin\theta - \sigma_2 \sin\theta \cos\theta - \tau \cos^2\theta + \tau \sin^2\theta$

$= (\sigma_1 - \sigma_2) \sin\theta \cos\theta - \tau (\cos^2\theta - \sin^2\theta)$

$\therefore \sigma_t = \frac{(\sigma_1 - \sigma_2)}{2} \sin 2\theta - \tau \cos 2\theta$

position of principle plane

To know the position of principle plane equating tangential stress is equal to 'zero'.

$\sigma_t = 0$

$\left(\frac{\sigma_1 - \sigma_2}{2}\right) \sin 2\theta - \tau \cos 2\theta = 0$

$\left(\frac{\sigma_1 - \sigma_2}{2}\right) \sin 2\theta = \tau \cos 2\theta$

$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$

$FC = \pm\sqrt{(2\tau)^2 + (\sigma_1 - \sigma_2)^2}$

$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(2\tau)^2 + (\sigma_1 - \sigma_2)^2}}$

$\cos 2\theta = \pm \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$

position of principle stresses

case-1:

principle stress

$\therefore \sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cos 2\theta + \tau \sin 2\theta$

$\therefore \sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\left(\frac{(\sigma_1 - \sigma_2)}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}\right) + \tau\left(\frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}\right)$

$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \frac{(\sigma_1 - \sigma_2)^2}{2\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}} + \frac{2\tau^2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$

$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \frac{(\sigma_1 - \sigma_2)^2 + 4\tau^2}{2\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$

$= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}{2}$

$\therefore \sigma_n = \frac{(\sigma_1 + \sigma_2) + \sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}{2}$

Case-2:

$\therefore \sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\left(\frac{-(\sigma_1 - \sigma_2)}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}\right) - \frac{2\tau^2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$

$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) - \frac{(\sigma_1 - \sigma_2)^2}{2\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}} - \frac{2\tau^2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$

$= \frac{\sigma_1 + \sigma_2}{2} - \frac{(\sigma_1 - \sigma_2)^2 + 4\tau^2}{2\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$

$= \frac{\sigma_1 + \sigma_2}{2} - \frac{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}{2}$

$\therefore \sigma_n = \frac{(\sigma_1 + \sigma_2) + \sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}{2}$

$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2} = \frac{\text{max p.s} - \text{min p.s}}{2}$

Examples

* At a point within a body, it is subjected to mutually perpendicular tensile stresses are 80 N/mm² & 40 N/mm². Each of the above stresses are accompained by a shear stress of 60 N/mm². Determine the normal stresses, tangential stress and resultant stresses on an oblique plane which is making an angle 45° with the major principle axis.

Sol:

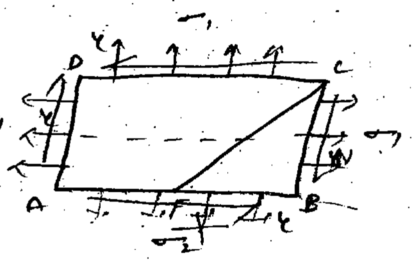
Given data;

$\sigma_1 = 80\ N/mm^2$

$\sigma_2 = 40\ N/mm^2$

$\tau = 60\ N/mm^2$

$\theta = 45^\circ$



normal stress,

$$\sigma_n = \left(\frac{\sigma_1+\sigma_2}{2}\right) + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta + \tau \sin 2\theta$$

$$= \left(\frac{80+40}{2}\right) + \left(\frac{80-40}{2}\right)\cos 2(45^\circ) + 60\sin(2\cdot 45^\circ)$$

$\therefore \sigma_n = 120\ N/mm^2$

$$\therefore \sigma_t = \left(\frac{\sigma_1 + \sigma_2}{2}\right)\sin 2\theta + \tau \cos 2\theta$$

$$= \left(\frac{80-40}{2}\right)\sin 2(45^\circ) + 60 \cos 2(45^\circ)$$

$\therefore \sigma_t = 20\ N/mm^2$

$\sigma_R = 121.65\ N/mm^2$

* Rectangular block is subjected to a tensile stress of 110 N/mm² one one plane and a tensile stress of 47 N/mm² which are mutually ⊥r. Each of the above stresses are accompained by a shear stress of 63 N/mm² and that are associated with formal tensile stress tends to rotate the block anticlockwise. Find the direction and magnitude of each principle stress and also find magnitude of greater shear stress.

Sol:

Given data,

$\sigma_1 = 110\ N/mm^2$

$\sigma_2 = 47\ N/mm^2$

$\tau = 63\ N/mm^2$

Case-1:

The principle stress are "zero"

$\sigma_t = 0$

$$\left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta - \cos 2\theta = 0$$

$$\tan 2\theta = \frac{2\tau}{\sigma_1-\sigma_2}$$

$$= \frac{2(63)}{110-47}$$

$\tan 2\theta = 2.0$

$2\theta = 63^\circ 26'$

$\theta = 31^\circ 43'$

$$\therefore \sigma_n = \left(\frac{\sigma_1+\sigma_2}{2}\right) + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta + \tau \sin 2\theta$$

$$= \left(\frac{110+47}{2}\right) + \left(\frac{110-47}{2}\right)\cos 2(31^\circ 43') + 63 \sin 2(31^\circ 43')$$

To obtain the max shear stress $\frac{d}{d\theta}(\sigma_t) = 0$

$$\frac{d}{d\theta}\left[\left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta\right] - \tau \frac{d}{d\theta}\cos 2\theta = 0 \quad \left[\because \frac{d}{d\theta}\sin 2\theta = 2\cos 2\theta\right]$$

$$\left(\frac{\sigma_1-\sigma_2}{2}\right) 2\cos 2\theta + 2\tau \sin 2\theta = 0$$

$$2\left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta = -2\tau \sin 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = -\left(\frac{\sigma_1-\sigma_2}{2\tau}\right)$$

$$\tan 2\theta = -\left(\frac{\sigma_1-\sigma_2}{2\tau}\right)$$

$$= -\left(\frac{110-47}{2 \times 63}\right)$$

$$\tan 2\theta = \frac{47-110}{2 \times 63}$$

$$\tan 2\theta = -0.5$$

$$2\theta = -26^\circ 33'$$

$$\theta = -13^\circ 16'$$

The Graphical representation of determining Normal stresses, tangential stresses and Resultant stresses in mohr's circle method.

The 3 cases which are analyzed by Mohr's circle method are,

1. Body subjected to mutually perpendicular ~~lens~~ stress with unequal intensities.
2. Body subjected to mutually perpendicular stress with unlike and unequal stresses.
3. Member subjected to 2 mutually ⊥r tensile stresses accompainished by shear stress.

$\cos 2\theta = \frac{OE}{OD}$

$OE = OD\cos 2\theta$

Body subjected to 2 mutually perpendicular stresses with unequal intensities

let σ_1 = major tensile stress

σ_2 = minor tensile stress

θ = angle made by the oblique plane with the minor axis.

ϕ = obliquity.

→ let AB be any line which represents the major tensile stress and BC be the any point on AB for which AC represents the minor tensile stress.

→ BC as dia draw a circle through the points B and C bisect line BC obtaining center as 'O' such that OC = OB radius of circle

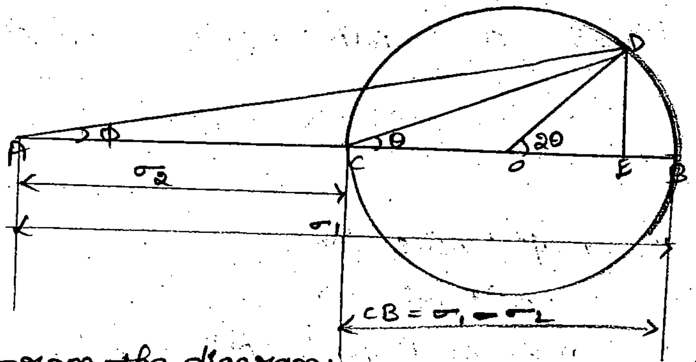
→ let D be any point on the circle which is making an angle θ with 'C'.

→ Draw a ⊥r line from D to meet at E on line BC
→ Join OD such that OB = OC = OD
→ Join CD and AD.
→ The length AE represents the normal stresses.
→ The length DE represents the tangential stresses
→ The length AD represents the resultant stresses.

Proof:

* AE = Normal stress

we have $\sigma_n = \frac{\sigma_1+\sigma_2}{2} + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta$



From the diagram:

$AE = AO + OE$

$= AO + OD\cos 2\theta$

$= AO + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta$

$= AC + OC + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta$

[∵ from Δ^{le} ODE, $\cos 2\theta = \frac{OE}{OD}$, $OE = \cos 2\theta\, OD$, $OB = OC = OD = \frac{\sigma_1-\sigma_2}{2}$]

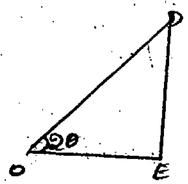
$= \sigma_2 + \frac{\sigma_1-\sigma_2}{2} + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta$

$= \frac{2\sigma_2+\sigma_1-\sigma_2}{2} + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta$

$= \frac{\sigma_1+\sigma_2}{2} + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta$

* DE = Tangential stresses

we know that $\sigma_t = \frac{\sigma_1-\sigma_2}{2}\sin 2\theta$



from Δ^{le} ODE, $\sin 2\theta = \frac{DE}{OD}$

$DE = OD\sin 2\theta$

$DE = \left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta$

* AD = Resultant stress

Example

The tensile stresses at a point across 2 mutually ⊥r plane are 120 N/mm² and 60 N/mm². Determine Normal stress, tangential stress & resultant stress on an oblique plane which is making an angle 30° with the minor principle axis.

Sol:

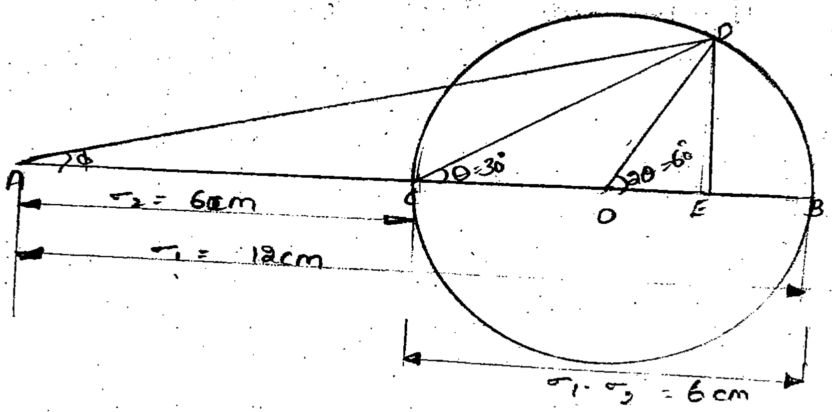
Given data:

$\sigma_1 = 120\ N/mm^2$

$\sigma_2 = 60\ N/mm^2$

$\theta = 30°$ with minor principle axis.

let 1cm = 10 N/mm²



procedure :

→ let AB = 12cm be a line which represents major tensile stress. And c be a point on AB line of AC = 6cm which represents the minor tensile stress.

→ BC = 6m as dia a circle through a point B and c bisect the line BC uptairing the centre 'o' such that OC = OB = 3cm radius.

→ let draw a line from point 'c' making angle 30° on circle. let it be D.

→ Draw a ⊥r line from D to meet at E on line BC.

→ Join the OD and CD and AD.

→ The length AE is normal stress.

→ The length DE is tangential stress.

→ The length AD is resultant stress.

* normal stress σ_n = 10.5 cm = 105 N/mm²

* tangential stress σ_t = 2.5 cm ≃ 25 N/mm²

* The resultant stress = 10.7 cm ≃ 107 N/mm²

2. The tensile stresses ... are 150 N/mm² & 100 N/mm². Determine the Normal & tangential and resultant stresses on an oblique plane which is making an angle 20° with the minor axis.

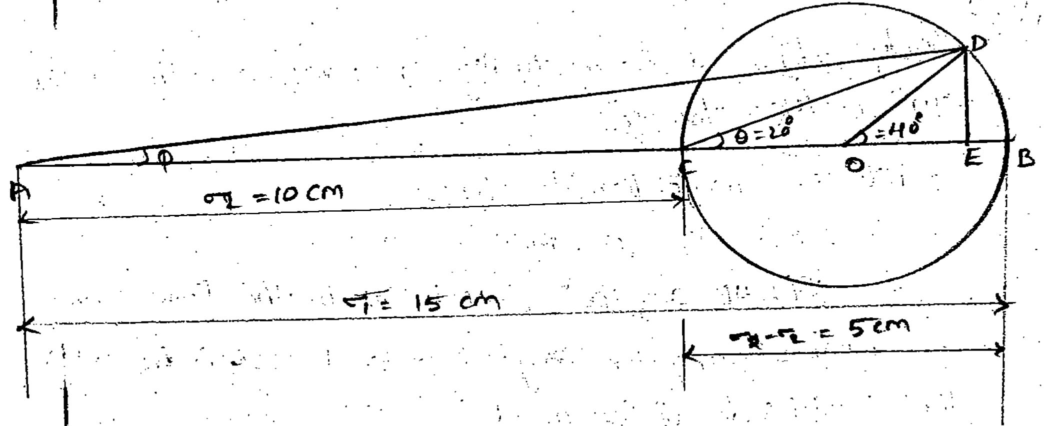
Sol: Given data

σ_1 = 150 N/mm² = 15cm

σ_2 = 100 N/mm² = 10cm

θ = 20° with minor

Scale: 1cm = 10 N/mm²



procedure

→ draw the major principle axis AB = 15cm and c be a point on AB line of AC = 10cm which represents the minor tensile stress.

→ BC = 5cm dia a circle through a point B and c bisect the line BC uptairing the centre 'o' such that OC = OB = 2.5cm.

→ let draw a line from 'c' making angle 20° on circle, let it be D

→ Draw a $\perp r$ from D to meet at E on BC line

→ join the ~~OD~~ AD and DC

→ The length of AE is normal stress

→ The length of DE is tangential stress.

→ The length of AD is resultant stress.

* The normal stress $= 14.4$ cm $\simeq 144$ N/mm^2
* The tangential stress $= 1.6$ cm $\simeq 16$ N/mm^2
* the resultant stress $= 14.4$ cm. $\simeq 144$ N/mm^2

7/12/18

Body subjected to mutually $\perp r$ stresses with unequal and unlike stresses

let σ_1 = major tensile stress

σ_2 = minor compressive stress.

→ let AB any point and AB be the line which is representing the major tensile stress (σ_1) towards the right side of point A.

→ let, C be the any point towards left side of point A which representing the 'σ_2' minor principle Compressive stress.

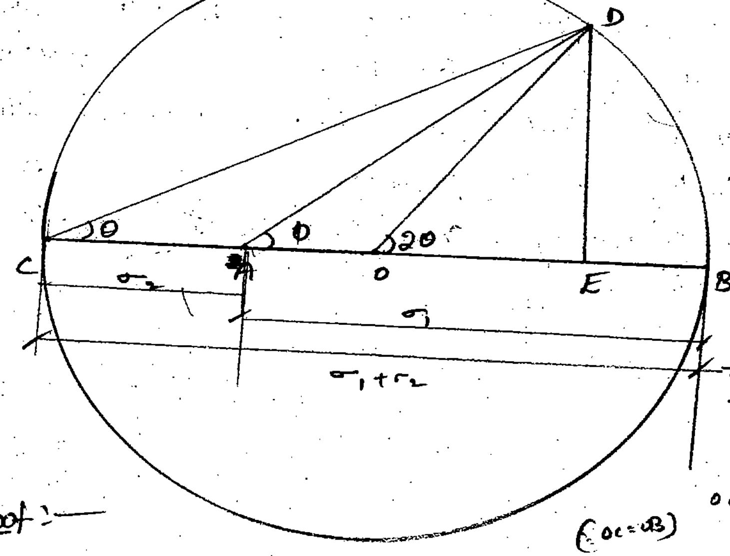
→ Bisect the BC line at 'O' as center.

→ Draw a circle OC as radius and 'O' as center.

→ let 'D' be the any point on the circle which is making an angle 'θ' with the minor axis and join CD

→ Draw a $\perp r$. line from 'D' to meet line BC at 'E'.

→ Now join OD and AD.



from fig
$AB = \sigma_1$
$AC = \sigma_2$
$BC = \sigma_1+\sigma_2$
$OC + OB = \sigma_1+\sigma_2$
(OC = OB)
$2OC = \sigma_1+\sigma_2$
$OD = OB = OC = \dfrac{\sigma_1+\sigma_2}{2}$

proof :—

from fig,

1. AE = Normal stress

$\therefore AE = AO + OE$

$= AO + OD\cos 2\theta$

$= AO + \left(\dfrac{\sigma_1+\sigma_2}{2}\right)\cos 2\theta$

$= \left(\dfrac{\sigma_1-\sigma_2}{2}\right) + \left(\dfrac{\sigma_1+\sigma_2}{2}\right)\cos 2\theta$

$= \left(\dfrac{\sigma_1+(-\sigma_2)}{2}\right) + \left(\dfrac{\sigma_1-(-\sigma_2)}{2}\right)\cos 2\theta$

$\therefore AE = \sigma_n$

from Δ^{le} OED,
$OE = OD\cos 2\theta$

$\therefore AO = CO - AC$
$= \dfrac{\sigma_1+\sigma_2}{2} - \sigma_2$
$= \dfrac{\sigma_1-\sigma_2}{2}$

2. DE = tangential stress

$= OD\sin 2\theta$

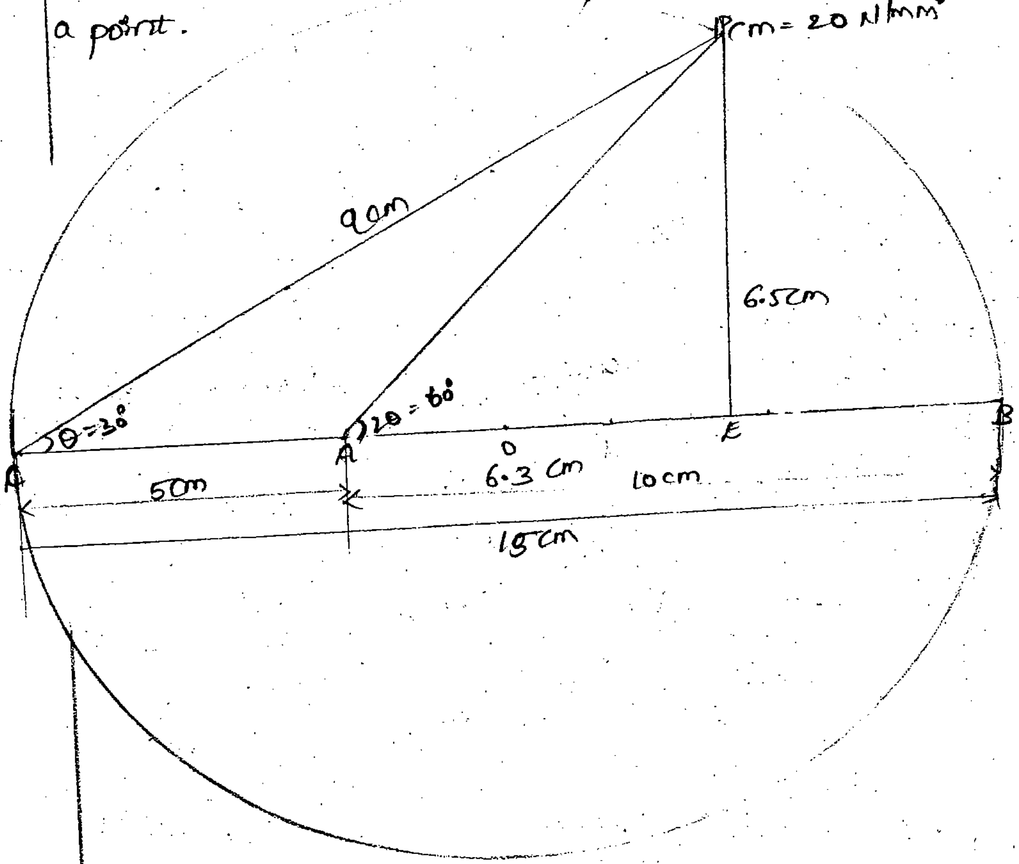
$= \left(\dfrac{\sigma_1+\sigma_2}{2}\right)\sin 2\theta$

$= \left(\dfrac{\sigma_1-(-\sigma_2)}{2}\right)\sin 2\theta$

$\therefore DE = \sigma_t$

from Δ^{le} OED
$OE = OD\sin 2\theta$

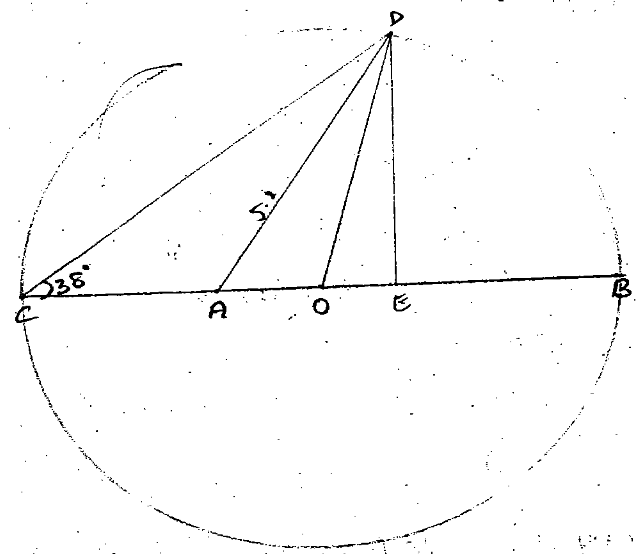
1. The stress at a point in a bar are 200 N/mm² tensile and 100 N/mm² compression. Determine the resultant stress in magnitude & direction on a plane inclined at 60 to the axis of major stresses. Also determine the max intensity of shear stress in the material at a point.



θ = 60°

2. The stress at a point in a bar are 150 ... and 75 N/mm² compression. Determine the normal stress, tangential stress, resultant stresses and obliquity on an inclined plane which is making an angle 38° with the minor axis.

Scale 1cm = 25 N/mm²



AE = σ_n = 2.7 cm

DE = τ = 4.4 cm

AD = σ_R = 5.2 cm

8/10/18

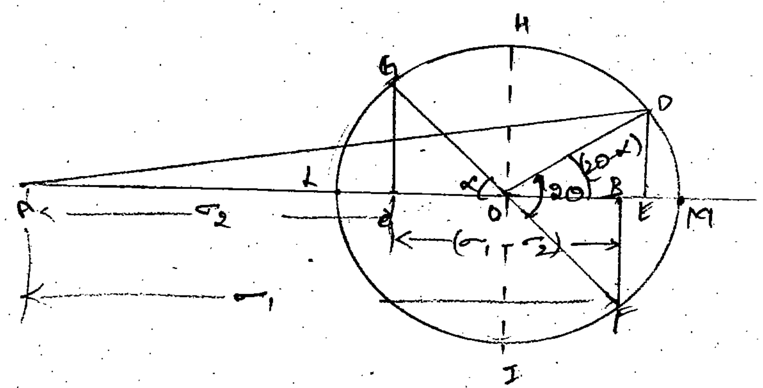
Body subjected to tensile on 2 mutually $\perp r$ planes accompainshed with simple shear

Let σ_1 = Major tensile stress

σ_2 = Minor tensile stress

τ = Shear stress

θ = angle made by oblique with the axis of minor stress.



→ Let AB be any line which is representing major tensile stress draw right side of point and AC be any line drawn towards point A. so that AC is minor tensile stress.

→ Draw a $\perp r$ line at point C towards and at point B down wards, so that the $\perp rs$ CG and FB represent the shear stresses.

→ Join points G and F, the line FG biseds the line BC at point 'O' so that OC equals to OB.

→ OG as radius 'O' as centre draw a circle.

→ An oblique plane making an angle θ with axis of minor stresses.

→ It makes an angle of 2θ wrt σ. the oblique plane meets the circle at point D.

→ Draw a $\perp r$ from D so that it meets line AB at point E. Join AD.

→ The length AE represents the normal stresses, the length DE Tangential stress, AD represents the resultant stresses. $\angle EAD$ represents the obliquity.

Proof:

AE = Normal stress

$= AO + OE$

$= AC + OC + OE$

$= \sigma_2 + \frac{\sigma_1 - \sigma_2}{2} + OD\cos(2\theta - \alpha)$

$= \sigma_2 + \frac{\sigma_1 + \sigma_2}{2} + OD\cos 2\theta\cos\alpha + OD\sin 2\theta\sin\alpha$

$= \sigma_2 + \frac{\sigma_1 - \sigma_2}{2} + OF\cos\alpha\cos 2\theta + OF\sin\alpha\sin 2\theta$

$= \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cos\theta + \tau\sin 2\theta$

$= \sigma_n$

From Δ^{le} OED

$\cos(2\theta - \alpha) = \frac{OE}{OD}$

$OE = OD\cos(2\theta - \alpha)$

$\left[\because \cos(A-B) = \cos A\cos B + \sin A\sin B\right.$

$\left.\because \sin(A-B) = \sin A\cos B - \cos A\sin B\right]$

$\therefore DE = OE\sin(2\theta - \alpha)$

$= OE\sin 2\theta\cos\alpha - OE\cos 2\theta\sin\alpha$

$= OF\cos\alpha\sin 2\theta - OE\sin\alpha\cos 2\theta$

$= OB\sin 2\theta - BF\cos 2\theta$

$= OC\sin 2\theta - \tau\cos 2\theta$

$= \left(\frac{\sigma_1 - \sigma_2}{2}\right)\sin 2\theta - \tau\cos 2\theta$

$\therefore DE = \sigma_t$.

Max & Min normal stress

$AD = \sigma_n\ max$ = meets the point M

$AM = AO + OM$
$= AC + CO + OM$ (∴ OM represents the radius of circle)
$= AC + CO + OF$

→ To obtain max normal stress the point E should coincide with point M

AM represents the max σ_n

from $\triangle^{le}$ OBF
$OF^2 = OB^2 + FB^2$
$\therefore OF = \sqrt{OB^2 + FB^2}$

$AM = AO + OM$
$= AC + CO + OM$
$= \sigma_1 + \frac{\sigma_1 - \sigma_2}{2} + OF$
$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{OB^2 + FB^2}$

$$\sigma_n\ max = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

→ σ_n min, AL represents the min σ_n

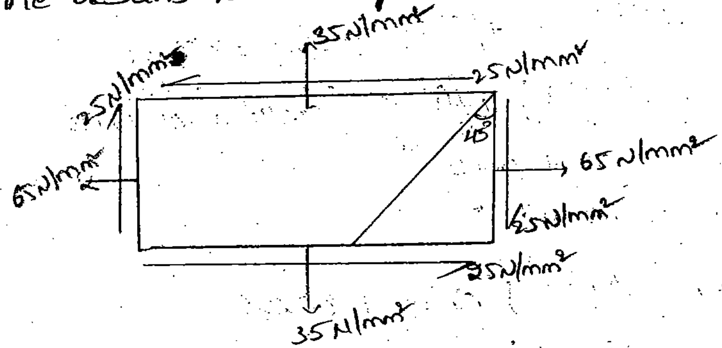
$AL = AO - OL$
$= AC + OC - OF$
$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

→ max shear stress

$OH = \tau_{max}$

$OH = OF$
$= \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

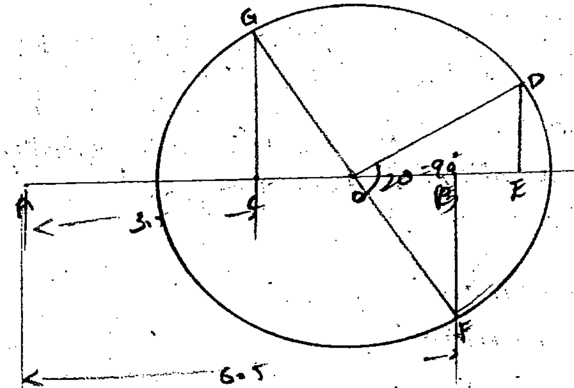
1. A point in a strained material is subjected to stresses as shown in fig using Mohr's circle method, determine the σ_n, σ_t, R and its obliquity on oblique plane which is making 45° with the plane of major stresses. Check the results with analytical method.



Sol) Given data.

$AB = \sigma_1 = 65\ N/mm^2$
$AC = \sigma_2 = 35\ N/mm^2$
$BF = \tau = 25\ N/mm^2$

Scale 1cm = 10 N/mm²
$\theta = 45°$
$2\theta = 90°$



Normal stress $\sigma_n = AE = 4.5\text{cm}$

tangential stress $\sigma_t = DE = 1.5\text{cm}$

Resultant stress $= 4.7\text{cm}$

By analytical method

$$\sigma_n = \left(\frac{\sigma_1+\sigma_2}{2}\right) + \left(\frac{\sigma_1-\sigma_2}{2}\right)\cos 2\theta + \tau \sin 2\theta$$

$$= \left(\frac{65+35}{2}\right) + \left(\frac{65-35}{2}\right)\cos 90^\circ + 25 \sin 90^\circ$$

$$= 50 + 0 + 25$$

$$\therefore \sigma_n = 75 \text{ N/mm}^2$$

$$\therefore \sigma_t = \left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta + \tau \cos 2\theta$$

$$= \left(\frac{65-35}{2}\right)\sin 90^\circ + \tau \cos 90^\circ$$

$$\therefore \sigma_t = 15 \text{ N/mm}^2$$

2. At a certain point in a strained material the intensities of stresses on 2 planes which are right angle to each other are 20 N/mm² and 10 N/mm² both are tensile and they are accompanied by simple shear of 10 N/mm². Find the principle stress by graphical method and also check the results analytically. Also determine the location of principle plane.

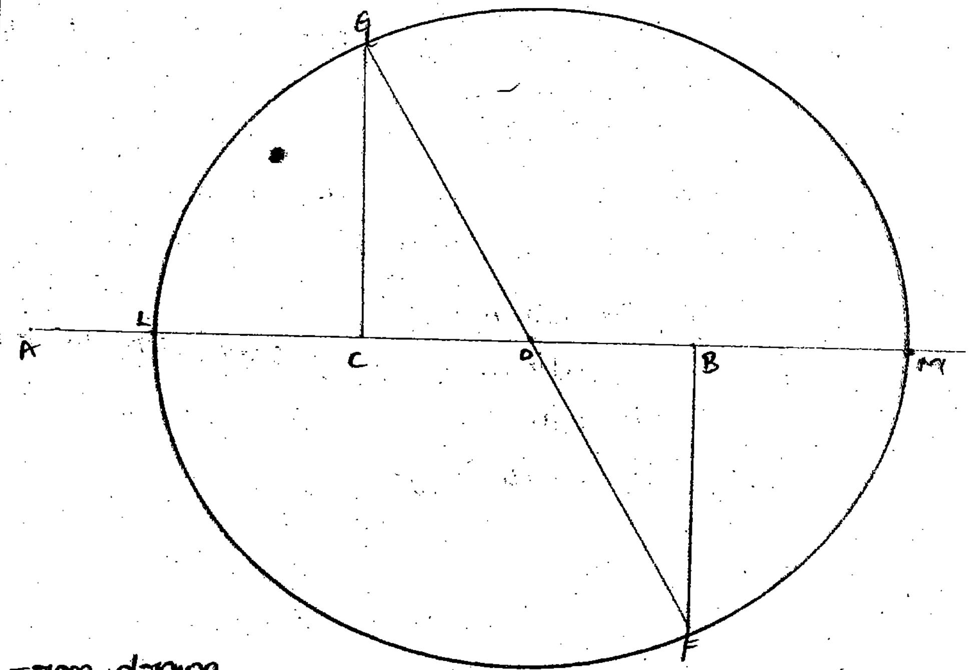
Sol: Given data,

$\sigma_1 = 20 \text{ N/mm}^2$

$\sigma_2 = 10 \text{ N/mm}^2$

$\tau = 10 \text{ N/mm}^2$

Let scale 1cm = 2N/mm²



From drawn

~~$AE = \sigma_n = 12.5\text{cm} = 25 \text{ N/mm}^2$~~

~~$DE = \sigma_t = 2.5\text{cm} = 5 \text{ N/mm}^2$~~

~~$AD = \sigma_R = 12.8\text{cm} = 25.6 \text{ N/mm}^2$~~

By analysis

The principle stresses, Max $AM = 13.1 \text{ cm} = 26.0 \text{ N/mm}^2$

Min $= AL = 1.8\text{cm} = 3$

3. At a certain point in a strained material the intensities of stresses on a planes which are right angle to each other are 80 N/mm² & 30 N/mm² both are tensile & they are accompainshed by a shear stress of 15 N/mm². Determine σ_n, σ_t, σ_R and its inclination on an oblique plane which is making 40° angle with the plane of major stresses. also determine principle stresses, magnitude of max shear stress.

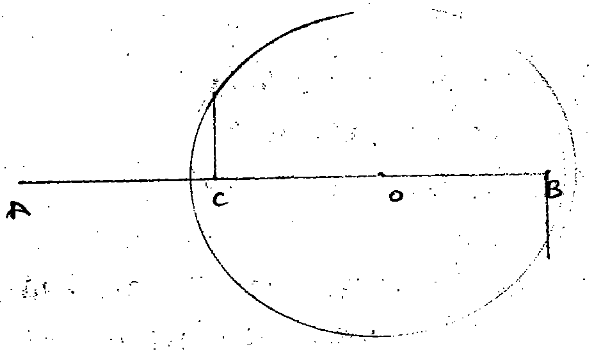
Sol: Given

$\sigma_1 = 80\ N/mm^2$

$\sigma_2 = 30\ N/mm^2$ Scale 1cm = 10 N/mm²

$\tau = 15\ N/mm^2$.

$\theta = 40°$ with major



AE = σ_n = 7.4cm = 74 N/mm²

DE = σ_t = 2.1cm = 21 N/mm²

AD = σ_R = 7.7cm = 77 N/mm²

AM = major principle stress = 8.4 = 84 N/mm²

AK = Min p.s = ...

Max shear stress = 3cm = 6 N/mm²

$$\tan\phi = \frac{\sigma_t}{\sigma_R}$$

$\phi = 15.84$

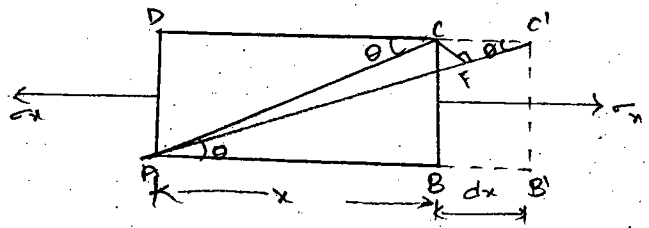
$\phi = 15° 50'$

Strain analysis:

1. Body subjected to stress in x-direction then strain on a oblique plane due to stress σ_x.

→ Consider a rect^r bar ABCD subjected to stress σ_x in x-direction as shown in fig.

→ Due to the stress σ_x their is an increase in length dx..



Strain in x direction on AB plane due to σ_x is given by, $e_x = \frac{\text{change length}}{\text{original length}}$

$$e_x = \frac{AB' - AB}{AB}$$

$$= \frac{BB'}{AB}$$

$$e_x = \frac{dx}{x}$$

Here dx is chain in length due to stress σ_x

$\therefore dx = x \cdot e_x$

Now, strain on an oblique plane is AC is given by, $e = \frac{\text{change in length}}{\text{original length}}$

$$e = \frac{AC' - AC}{AC}$$

But $AC' = AF + FC'$

$AF = AC$

$$\therefore e = \frac{\cancel{AC} + FC' - \cancel{AC}}{AC}$$

$$e = \frac{FC'}{AC}$$

— from Δ^{le} $FC'C$

$FC' = CC' \cos\theta$

$$\therefore e = \frac{CC' \cos\theta}{AC}$$

$$= \frac{dx \cdot \cos\theta}{AC}$$

$$\therefore e = \frac{x \cdot e_x \cos\theta}{AC}$$

$$= \frac{AB \cdot e_x \cdot \cos\theta}{AC}$$

— from Δ^{le} ACC'

$AB = AC \cos\theta$

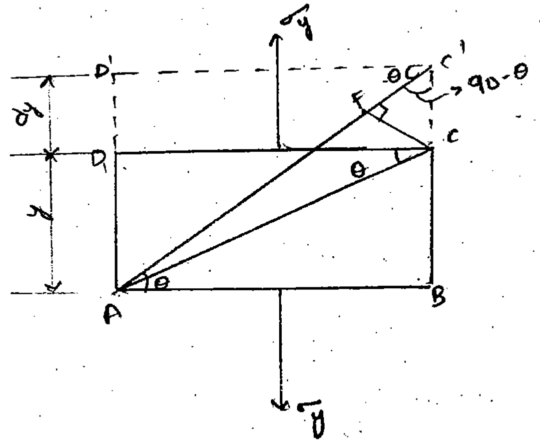
$$e = \frac{AC \cos\theta \cdot e_x \cos\theta}{AC}$$

$$\therefore e = e_x \cos^2\theta$$

2. strain on oblique due to stress in y-direction.

→ Consider a rectangular bar ABCD subjected to stress σ_y

→ Due to the stress σ_y their is an increase in width.



$\rightarrow e_y$ = strain in y direction on AD plane or BC plane

$$e_y = \frac{\text{change in length}}{\text{original length}}$$

$$= \frac{AD' - AD}{AD}$$

$$= \frac{DD'}{AD}$$

$$\therefore e_y = \frac{dy}{y}$$

Here dy is the chain in width ~~due to~~ stress σ_y

so, $\quad dy = e_y . y$

e = strain on an oblique plane AD

$$= \frac{\text{change in length}}{\text{original length}}$$

$$= \frac{AC' - AC}{AC}$$

$$= \frac{AF + FC' - AC}{AC}$$

$$e = \frac{FC'}{AC} = \frac{CC' \sin\theta}{AC}$$

$AC' = AF + FC'$

$AC = AF$

From Δ^{le} FCC'

$FC' = CC' \sin\theta$

$$e = \frac{\ldots}{AC}$$

$$e = \frac{y . e_y \sin\theta}{AC}$$

$$\therefore e = \frac{AD . e_y \sin\theta}{AC}$$

from ACD

$AD = AC \sin\theta$

$$= \frac{AC \sin\theta \; e_y \sin\theta}{AC}$$

$$\therefore e = e_y \sin^2\theta$$

14/12/18

3. Strain on a oblique plane when body is subjected to simple shear

Consider a rectangular beam ABCD as shown in fig which is subjected to shear stress.

The strain developed in plane BC denoted by ϕ can be given as, $\tan\phi = \frac{CC'}{BC}$

where CC' is the deformation and when ϕ is very small neglecting tan,

$$\therefore \phi = \frac{CC'}{BC}$$

$$CC' = \phi BC$$

$$CC' = \phi AC \sin\theta.$$

from Δ^{le} ABC

$BC = AC \sin\theta$

Consider an oblique plane which is making an angle θ with the axis of minor stresses there will be deformation in oblique plane also due to the shear stress.

$\therefore$ The strain developed on oblique plane denoted by e is given by $= \frac{\text{change in length}}{\text{original length}}$

$$e = \frac{AC' - AC}{AC}$$

$$e = \frac{AC + FC' - AC}{AC}$$

from Δ^{le} FCC'

$FC' = CC'\cos\theta$

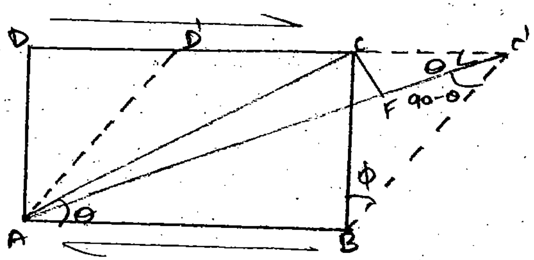
$$= \frac{FC'}{AC}$$

$$= \frac{CC'\cos\theta}{AC}$$

$$= \frac{\phi\, AC\sin\theta\cos\theta}{AC}$$

$$= \phi\sin\theta\cos\theta$$

$$\therefore e = \phi/2 \sin 2\theta$$



4. Strain on an oblique plane when body is subjected to stress in x-direction, stress in y-direction and these stress are accomplished by shear stress

Consider a rectangular bar ABCD which is subjected to stress in x-direction, stress in y-direction and simple shear.

The strain in oblique plane is given by,

$e =$ strain due stress in σ_x + strain due to stress in y + strain due to shear

$$= e_x\cos^2\theta + e_y\sin^2\theta + \frac{\phi}{2}\sin 2\theta$$

25

$$= e_x\left(\frac{1+\cos 2\theta}{2}\right) + e_y\left(\frac{1-\cos 2\theta}{2}\right) + \frac{\phi}{2}\sin 2\theta$$

$$= \frac{e_x}{2} + \frac{e_x\cos 2\theta}{2} + \frac{e_y}{2} - \frac{e_y\cos 2\theta}{2} + \frac{\phi}{2}\sin 2\theta$$

$$\therefore e = \frac{e_x+e_y}{2} + \left(\frac{e_x-e_y}{2}\right)\cos 2\theta + \frac{\phi}{2}\sin 2\theta$$

Max and min principle strains

These are can be obtained when,

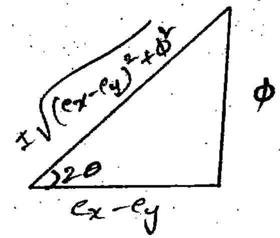
$$\frac{de}{d\theta} = 0$$

$$\frac{d}{d\theta}\left[\left(\frac{e_x+e_y}{2}\right) + \left(\frac{e_x-e_y}{2}\right)\cos 2\theta + \frac{\phi}{2}\sin 2\theta\right] = 0$$

$$\left[\left(\frac{e_x + e_y}{\cancel{2}}\right)\cancel{2}(-\sin 2\theta) + \frac{\phi}{\cancel{2}}(\cancel{2}\cos 2\theta)\right] = 0$$

$$(e_x - e_y)\sin 2\theta = \phi\cos 2\theta$$

$$\tan 2\theta = \frac{\phi}{e_x - e_y}$$



from the Δ^{le},

$$\sin 2\theta = \pm\frac{\phi}{\sqrt{(e_x-e_y)^2+\phi^2}}$$

$$\cos 2\theta = \pm\frac{e_x-e_y}{\sqrt{(e_x-e_y)^2+\phi^2}}$$

Max principle strain is obtained when $\sin 2\theta$ & $\cos 2\theta$ are taken as +ve i.e., $\sin 2\theta = \pm\frac{\phi}{\sqrt{(e_x-e_y)^2+\phi^2}}$

$$\cos 2\theta = \frac{\phi}{\sqrt{(e_x-e_y)^2+\phi^2}}$$

$$\therefore e = \left(\frac{e_x+e_y}{2}\right) + \left(\frac{e_x-e_y}{2}\right)\left(\frac{e_x-e_y}{\sqrt{(e_x-e_y)^2+\phi^2}}\right) + \frac{\phi}{2}\left(\frac{\phi}{\sqrt{(e_x-e_y)^2+\phi^2}}\right)$$

$$= \left(\frac{e_x+e_y}{2}\right) + \frac{(e_x-e_y)^2}{2\sqrt{(e_x-e_y)^2+\phi^2}} + \frac{\phi^2}{2\sqrt{(e_x-e_y)^2+\phi^2}}$$

$$= \frac{e_x+e_y}{2} + \frac{(e_x-e_y)^2+\phi^2}{2\sqrt{(e_x-e_y)^2+\phi^2}}$$

$$= \frac{e_x+e_y}{2} + \frac{\sqrt{(e_x-e_y)^2+\phi^2}}{2}$$

$$\therefore e = \frac{(e_x+e_y)+\sqrt{(e_x-e_y)^2+\phi^2}}{2}$$

For min principle strain is obtained when $\sin 2\theta$ and $\cos 2\theta$ are -ve i.e.,

$$\sin 2\theta = -\frac{\phi}{\sqrt{(e_x-e_y)^2+\phi^2}}$$

$$\cos 2\theta = -\frac{(e_x-e_y)}{\sqrt{(e_x-e_y)^2+\phi^2}}$$

$$\therefore e = \left(\frac{e_x+e_y}{2}\right) + \left(\frac{e_x-e_y}{2}\right)\left(-\frac{(e_x-e_y)}{\sqrt{(e_x-e_y)^2+\phi^2}}\right) + \frac{\phi}{2}\left(-\frac{\phi}{\sqrt{(e_x-e_y)^2+\phi^2}}\right)$$

$$= \frac{e_x+e_y}{2} - \frac{(e_x-e_y)^2}{2\sqrt{(e_x-e_y)^2+\phi^2}} - \frac{\phi^2}{2\sqrt{(e_x-e_y)^2+\phi^2}}$$

$$\therefore e = \frac{(e_x+e_y)-\sqrt{(e_x-e_y)^2+\phi^2}}{2}$$

Theory of failures

→ When an external load is applied on a body the body to which stresses & strains are produced in the body with the stresses are directly propotional to strains in elastic limit, this means after removal of load body regains its original position. That is their is no permanent deformation in the body.

If these stresses produced in the body due to application of load are beyond the Elastic limit then their will be a permanent deformation in the body.

This means if the load is removed the body will not regain its shape.

Due to this permanent deformation the body is said to be under failure.

Let us consider the failures occuring in a bar in a simple tensile test.

The tensile stress developed in the body which are directly propotional to tensile strain within the Elastic limit, this means upto Elastic limit these tensile stresses have a definite value

Beyond the Elastic limit if their is in increase in tensile stresses the failure of the bar takes place. The failure may be due to the following cases.

1. Max. principle stresses.
2. Max. principle ~~stresses~~ strain.
3. Max ~~stress~~ strain energy
4. Max shear stress
5. Max shear strain energy

Max. principle stress:

→ These theory is also known as rankines theory.

→ According to this theory the failures is occures when the max principle stresses in the complex stress system attains the value of max stress at the Elastic limit in simple tension,

or

when the min. principle stresses, i.e, max compressive stress attains the max stress at the Elastic limit in simple compression.

Thus, in this theory the max. principle stresses and min. principle stresses are the major criteria of failure.

Let, σ_x, σ_y, τ be the direct stresses and shear stresses on given plane and

σ_1 = max. principle stress, i.e,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

If, the max. principle stress (σ_1) is the design criteria then the max. principle stresses should not exceed the permisable stresses (σ_1^*).

Hence $\sigma_x = \sigma_1$, where σ_1^* is the permeasiable stresses, then $\sigma_1 = \frac{\sigma_1^*}{F.O.S}$

From this, Factor of safety = $\frac{\sigma_1^*}{\sigma_1}$

20/12/1? Examples

1. principle stresses in a cast iron body are 40mpa tensile and 90 mpa compression. The 3rd principle stress is being zero. determine the F.O.S based on elastic limit. If the criteria of failure is principle stress theory, the elastic limit in simple tensile is 80 MPa and in simple compression is 450 MPa for cast iron

Sol:

Given data,

let, the principle stress $\sigma_t = 40$ Mpa (+)

$\sigma_c = 90$ Mpa (−)

$\Rightarrow F.O.S = \frac{\sigma_1^*}{\sigma_1} = ?$

Elastic limit in simple tensile $\sigma_t^* = 80$ MPa

$\sigma_c^* = 450$ MPa

$$F.O.S = \frac{\sigma_t^*}{\sigma_t} = \frac{80}{40} = 2$$

$$F.O.S = \frac{\sigma_c^*}{\sigma_c} = \frac{450}{90} = 5$$

Max principle strain theory

This theory also known as Saint - venat theory.

According to this theory the failure of a material occurs when the max. principle strains reaches the value of max. strain at elastic limit in Simple tension or when the min principle strains reaches the value of max. strain at elastic limit in simple ~~tension~~ compression.

let, σ_1, σ_2 and σ_3 are the principle stresses and e_1, e_2 and e_3 are the principle strains due to principle strains.

$\therefore e_1, e_2, e_3$ can be given as,

$$e_1 = \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2 + \sigma_3)\right]$$

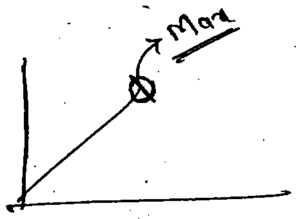
$$e_2 = \frac{1}{E}\left[\sigma_2 - \mu(\sigma_3 + \sigma_1)\right]$$

$$e_3 = \frac{1}{E}\left[\sigma_3 - \mu(\sigma_1 + \sigma_2)\right]$$

Now let $e_t^* =$ max. strain at elastic limit

e_t^* can be written as,

$$e_t^* = \frac{1}{E}\sigma_t^*$$



$$e_1 \geq e_t^*$$

$$\frac{1}{E}\left[\sigma_1 - \mu(\sigma_2 + \sigma_3)\right] \geq \frac{1}{E}\sigma_t^*$$

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) \geq \sigma_t^*$$

$$e_3 < e_c$$

$$\frac{1}{E}\left[\sigma_3 - \mu(\sigma_1 + \sigma_2)\right] \geq \frac{1}{E}\sigma_c^*$$

$$\sigma_3 - \mu(\sigma_1 + \sigma_2) \geq \sigma_c^*$$

Example:

1. The principle stresses at a point are ~~200~~ 200 N/mm² tensile & 50 N/mm² compression. If the stress at Elastic limit in Simple tension is 200 N/mm². Determine whether the failure will occurs ~~and~~ not by using max. principle strain theory. Take poission's ratio as 0.3.

Given, $\sigma_1 = 200$ N/mm² (tensile)
$\sigma_2 = 100$ N/mm² (tensile)
$\sigma_3 = 50$ N/mm² (compression)
poissions ration $\mu = 0.3$

stress at Elastic limit in $\sigma_t^* = 200$ N/mm²

we know the relation,

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) \geq \sigma_t^*$$

$$200 - 0.3(100 - 50) \geq \sigma_t^* \; 200$$

$$185 \geq 200$$

$$\therefore 185 \ngeq 200$$

$\therefore$ Hence the max. principle stress > the max. Elastic limit, hence the s/c will not failure.

2. Determine the dia. of bolt which is subjected to an axial pull of 9 kN together with transfers shear force of 4.5 kN using
1. Max. principle stress theory
2. Max principle strain theory. Given factor of safety is 3. poisson's ratio is 0.3 and the stress at elastic limit 225 N/mm² in simple tension.

Sol: let axial pull $P = 9$ kN P → direct st.
Transverse shear force $F = 4.5$ kN F →

$F.O.S = 3$

$\mu = 0.3$

$\sigma_t^* = 225$ N/mm²

let σ be the direct stress due to axial pull

$$\sigma = \frac{P}{A} = \frac{9 \times 10^3}{\frac{\pi}{4} \times d^2}$$

$$\sigma = \frac{11459.15}{d^2}$$

let τ be the shear stress due to shear force.

$$\sigma = \frac{F}{A} = \frac{4.5 \times 10^3}{\frac{\pi}{4} d^2}$$

$$= \frac{5429.57}{d^2}$$

let σ_1, σ_2 be the principle stresses, then it is given as,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$\sigma_x = \frac{11459.15}{d^2}, \quad \sigma_y = 0$$

$$\tau = \frac{5729.57}{d^2}$$

$$\sigma_1 = \frac{11459.15}{2d^2} + \sqrt{\left(\frac{11459.15}{2d^2}\right)^2 + \left(\frac{5729.57}{d^2}\right)^2}$$

$$= \frac{11459.15}{2d^2} + \sqrt{\frac{32828029.68}{d^4} + \frac{32829972.38}{d^4}}$$

$$= \frac{5729.575}{d^2} + \cancel{\frac{5933.86}{d^2}} \quad \frac{8102.83}{d^2}$$

$$\therefore \sigma_1 = \cancel{\frac{138663.46}{d^2}} = \frac{13826.41}{d^2}$$

$$\sigma_2 = \frac{11459.15}{2d^2} - \sqrt{\left(\frac{11459.15}{2d^2}\right)^2 + \left(\frac{5729.57}{d^2}\right)^2}$$

$$\therefore \sigma_2 = \frac{-2373.26}{d^2}$$

$$\therefore \sigma_t = \frac{\sigma_t^*}{F.O.S}$$

$$= \frac{225}{3}$$

$$= 75 \text{ N/mm}^2$$

1. $\sigma_1 = \sigma_t$

$$\frac{13832.45}{d^2} = 75$$

$$d^2 = \frac{13832.45}{75}$$

$$d = 13.6 \text{ mm}$$

2. $\sigma_1 - \mu(\sigma_2 + \sigma_3) = \sigma_t$

$$\frac{13832.45}{d^2} - \left\{0.3\left[0 + \left(-\frac{2373.26}{d^2}\right)\right]\right\} = 75$$

$$\frac{13832.45}{d^2} + 0.3 \times \frac{2373.26}{d^2} = 75$$

$$\frac{14544.42 - \cancel{11459.19}}{d^2} = 75.3$$

$$d^2 = \frac{11459.19}{75.3}$$

$$d^2 = 193.15$$

$$d = 13.89 \text{ mm}$$

2) Max shear stress theory

→ According to this theory the failure of a material occurs when the max shear stress reaches the value max shear stress at elastic limit in a material in simple tension.

→ This theory is also known as Guest & Treca's theory.

→ The max shear stress can be given as

$= \frac{1}{2}$ [difference b/w max & min principle stresses]

→ Let σ_1, σ_2, and σ_3 are the principle stresses in a material, then the max shear stresses are

$$= \frac{1}{2}[\sigma_1 - \sigma_3]$$

→ Let, σ_t^*, σ_t^* and 0 are the max. principle stresses at elastic limit in simple tension.

Max. shear stress $= \frac{1}{2}[\sigma_t^* - 0]$

$$= \frac{1}{2}(\sigma_t^*)$$

→ For a failure dc the max. shear stresses are > the max. shear stress at elastic limit.

$$\therefore \frac{1}{2}[\sigma_1 - \sigma_3] \geq \frac{1}{2}(\sigma_t^*)$$

$$\boxed{[\sigma_1 - \sigma_3] \geq \sigma_t^*}$$

For design criteria, let σ_t are the permeasible stress, then the max-stresses in the material is equal to the permeasible stresses

$$\boxed{\therefore \sigma_1 - \sigma_3 = \sigma_t}$$

And the permeasible stresses are given by

$$\sigma_t = \frac{\sigma_t^*}{F.O.S}$$

Example:

1. The p-stresses at a point in a material are 200 N/mm² tensile, 100 N/mm² tensile and 50 N/mm² compressive. If the max. stress at elastic limit in simple tension 200 N/mm² then determine whether the failure of a material will occurs or not according to the max shear stress theory.

Sol: Given data,

$\sigma_1 = 200$ N/mm² tensile

$\sigma_2 = 100$ N/mm² tensile

$\sigma_3 = 50$ N/mm² compression.

$\sigma_t^* = 200$ N/mm² at elastic limit.

We know the condition,

$$\sigma_1 - \sigma_3 \geq \sigma_t^*$$

$$200 + 50 \geq 200$$

$$250 \geq 200$$

The max. principle shear stresses are greater then the max shear stresses at elastic limit. Hence the material can fail.

2. At a s/c of mild steel shaft the max torque is 8473.5 Nm and the max. bending momentum is 50625 Nm and dia of shaft is 90 mm & the stresses at elastic limit in simple tension of the shaft is 220 N/mm². Determine whether the failure of a material will occur or not according to the Guest and treca's theory.

A. max torque of shaft $\tau = 8473.5$ Nm

$\tau \rightarrow \tau$

$M \rightarrow \sigma$

dia of shaft $d = 90$ mm $= 0.09$

$\frac{M}{I} = \frac{f}{y}$

max. Bending moment $M = 50625$ Nm

$\sigma_t^* = 220$ N/mm²

The condition for failure of a material is

$$[\sigma_1 - \sigma_3] \geq \sigma_t^*$$

we know that the max. principle stress formula

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

From bending moment equation w.k.t

$$\frac{M}{I} = \frac{f}{y} \qquad (f = \sigma)$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\frac{N\,m^2}{m^4}$$

$$\frac{Nm \times m}{\frac{\pi}{64}\, m^4} \; N/$$

$$\therefore \sigma = \frac{M \times y}{I} = \frac{50625 \times \frac{d}{2}}{\frac{\pi}{64} \times d^4} = \frac{50625 \times \frac{0.09}{2}}{\frac{\pi}{64} \times (0.09)^4}$$

$$= 31.729 \text{ MN/m}^2$$

$\tau = \frac{\pi}{16} * d^3 * \tau$

$8473.5 = \frac{\pi}{16} * (0.09)^3 * \tau$

$\tau = 59.2\ N/mm^2$

$\therefore \sigma_x = 70.729,\ \sigma_y = 0,\ \tau = 59.2$

$$\sigma_1 = \frac{70.729}{2} + \frac{1}{2}\sqrt{\left(\frac{70.729}{2}\right)^2 + 4(59.2)^2}$$

$= 104.32\ N/mm^2$

$$\sigma_3 = \frac{70.729}{2} - \frac{1}{2}\sqrt{\left(\frac{70.729}{2}\right)^2 + (4 * 59.2)^2}$$

$= -33.59\ N/mm^2$

$\therefore 104.32 + 33.059 \geq 220$

$137.91\ \cancel{144.7770.48} \geq 220$

Hence the material can not fail.

$\sigma_1 - \sigma_3 = \sigma_t$

$104.32 + 33.059 = \sigma_t$

$\sigma_t = 137.92$

$\text{F.O.S} = \frac{\sigma_t^*}{\sigma_t}$

$\text{F.O.S} = \cancel{220/144.07} \quad \frac{220}{137.92}$

$\text{F.O.S} = 1.59$

Max. Shear c.t

<u>Max. strain Energy theory :</u>

→ This theory is also known as haigh's theory.

→ According to this theory the failure of a material occurs when the max. strain energy reaches the value at max. strain energy at elastic limit.

→ let u is the strain energy and it is nothing but the total workdone by the given force in straining a material, it is given as.

$u = \frac{1}{2} * P * \delta L$ $\quad \therefore P = \sigma * A$

$= \frac{1}{2} * \sigma * A * L * e$ $\quad \therefore \delta L = L * e$

$= \frac{1}{2} * \sigma * e * A * L$

$u = \frac{1}{2} * \sigma * e * V$

$\therefore$ The energy per unit volume is given as,

$\frac{u}{V} = \frac{1}{2} * \text{stress} * \text{strain}$

let for a 3 dimensional element $\sigma_1, \sigma_2, \sigma_3$ are the principle stresses and e_1, e_2, e_3 are the corresponding principle strains.

Now, the total strain energy per unit volume in 3D material is given as,

$$\frac{u}{Vol} = \frac{1}{2} * \sigma_1 * e_1 + \frac{1}{2}\sigma_2 * e_2 + \frac{1}{2} * \sigma_3 * e_3$$

But $e_1 = \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)]$

$e_2 = \frac{1}{E}[\sigma_2 - \mu(\sigma_1 + \sigma_3)]$

$e_3 = \frac{1}{E}[\sigma_3 - \mu(\sigma_1 + \sigma_2)]$

$$\frac{u}{V} = \frac{1}{2E}\left[\sigma_1\left[\sigma_1-\mu(\sigma_2+\sigma_3)\right] + \sigma_2\left[\sigma_2-\mu(\sigma_3+\sigma_1)\right] + \sigma_3\left[\sigma_3-\mu(\sigma_1+\sigma_2)\right]\right]$$

$$\therefore \frac{u}{V} = \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]$$

let u^* be the strain energy at elastic limit then, the total strain energy at elastic limit per unit volume can be given as

$$\frac{u_t^*}{V} = \frac{1}{2} * \sigma_t^* * e_t^*$$

But w.k.t $e_t^* = \frac{\sigma_t^*}{E}$

$$\frac{u_t^*}{\text{unit } V} = \frac{1}{2E} * (\sigma_t^*)^2$$

For a failure material,

$$\frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right] \geq \frac{1}{2E}(\sigma_t^*)^2$$

$$\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right] \geq (\sigma_t^*)^2$$

Example:

1 The principle stresses at a point in a strained material are 200 N/mm² tensile, 100 N/mm² tensile, 50 N/mm² compression. If the stress at elastic limit is 200 N/mm² then determine whether the material will fail or not according to haigh's theory. $\mu = 0.3$.

Sol: Given data,

$\sigma_1 = 200$ N/mm² tensile

$\sigma_2 = 100$ N/mm² tensile

$\sigma_3 = 50$ N/mm² compressive

$\mu = 0.3$

$\sigma_t^* = 200$ N/mm²

$$\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1) \geq (\sigma_t^*)^2$$

$$200^2+100^2+50^2-2*0.3(200*100+100*50+50*200) \geq 200^2$$

$$52,500 - 300 \geq 40,000$$

$$49,500 \geq 40,000$$

The total strain energy per unit volume is greater than the strain energy. Hence the material can fail.

Max. Shear strain Energy theory:

This theory is also known as Mises-Henky's theory or Energy distortion theory.

According to this theory the failure of a material occurs when the total shear strain energy per unit volume reach the max. shear strain energy per unit volume at elastic limit in simple tension

→ let σ_1, σ_2, and σ_3 are the principle stresses then the total shear strain Energy per unit volume Can be given as.

$$\frac{1}{12C}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

let $\sigma_t^*, 0, 0$ are the principle stresses at Elastic limit for a uniaxial stress system then the total shear strain energy at Elastic limit Can be given as,

$$= \frac{1}{12C}\left[(\sigma_t^*-0)^2+(0-0)^2+(0-\sigma_t^*)^2\right]$$

$$= \frac{1}{12C}\left((\sigma_t^*)^2+(\sigma_t^*)^2\right)$$

$$= \frac{1}{12C}\left[2(\sigma_t^*)^2\right]$$

For a failure material the total shear strain Energy per unit volume is greater than or Equals to shear strain energy per unit volume at Elastic limit.

$$\frac{1}{12C}\left((\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^3+(\sigma_3-\sigma_1)^2\right) \geq \frac{1}{12C}\left[2(\sigma_t^*)^2\right]$$

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^3+(\sigma_3-\sigma_1)^2 \geq 2(\sigma_t^*)^2$$

let σ_t = permeasible stresses whis given by $\frac{\sigma_t^*}{F.O.S}$

For design consider the total shear strain Energy per unit volume is given Equal to permeasiable stresses.

$$\therefore (\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 = 2(\sigma_t)^2$$

1. The principle stresses at a point in strained material are 200 N/mm² tensile, 100 N/mm² tensile and 50 N/mm² compression. If the stress at Elastic limit is 200 N/mm², determine whether the failure of a material will occure or not according to Energy distotion theory. If their is no failure find the F.O.S.

Sol: Given data

$\sigma_1 = 200$ N/mm² (+ve)

$\sigma_2 = 100$ N/mm² (+ve)

$\sigma_3 = 50$ N/mm² (−ve)

$\sigma_t^* = 200$ N/mm²

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 \geq 2(\sigma_t^*)^2$$

$$95{,}000 \geq 80{,}000$$

For stress $95000 = 2(\cancel{200})(\sigma_t)^2$

$\sigma_t = 217.95$

$$F.O.S = \frac{\sigma_t^*}{\sigma_t} = \frac{200}{217.95} = 0.9176.$$